On Reissner's hypothesis

Historical proposals for a Machian unification of gravity and inertia

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1. Einstein's Machian intentions

2. Reissner's hypothesis

3. Gravinertial unification

4. A critical density cosmos

Einstein quote

- "the fact that the identity of the inertial and the gravitational mass has proved correct with such a remarkable accuracy seems to me to be one of the most important pointers for the development of the theory. The need to find a more profound explanation for that identity, and besides that also the view concerning the relativity of inertia advanced by Mach, was actually the motive that compelled me to devote myself to the problem of gravitation." (Einstein, 1996, p.228)

Figure 1: Albert Einstein

Two converging hypotheses

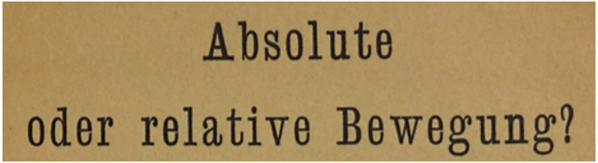
Mach's hypothesis:
inertia has a material origin in mass-interactions

Equivalence hypothesis:
"gravity and inertia are the same in their very essence" (Lehmkuhl, 2021)

Machian gravinertial unification:
by relativizing inertia we automatically get gravity as a side effect

Immanuel Friedlaender's footnote

Figure 2: Immanuel Friedlaender



- "However, it seems to me that the correct form of the law of inertia will only then have been found when relative inertia as an effect of masses on each other and gravitation, which is also an effect of masses on each other, have been derived on the basis of a unified law." (Friedlaender and Friedlaender, 1896)
- See Barbour and Pfister (1995) and Renn (2007) for translations of the text into English

What happened to Einstein's theory?

We can distinguish two separate hypotheses in Mach's works:[1]

Mach's principle:
inertia is required to have a physical origin

Mach hypothesis 1:
space is a physical medium responsible for inertia

Mach hypothesis 2:
physical space is an "abbreviated reference" to masses — which are the sources of inertial influences

Despite Einstein's original intentions, GR falls in this first category[2]

[1] See my paper: "Mach's principle and Mach's hypotheses" (Fay, 2024) for more context

[2] See Hoefer (1994) for a detailed discussion of Einstein's Machian intentions

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Who was Hans Reissner?

Figure 3: Hans and Josephine Reissner

- Pioneering German aeronautical engineer
- First person to build an all-metal aircraft (Reissner Canard)
- First to derive 'Reissner-Nördstrom metric' (Reissner, 1916)
- Published a pair of papers (Reissner, 1914, 1915) implementing Mach's idea that inertia is due to an interaction with masses

Necessity of gravitation

Über eine Möglichkeit die Gravitation als unmittelbare Folge der Relativität der Trägheit abzuleiten.

Von H. Reißner, Charlottenburg.

- "Mr. Einstein's equivalence hypothesis which asserts the mechanical and optical identity of an acceleration field with a field of constant gravity seems to imply the deeper meaning that gravity is also a resistance to acceleration."
- Reissner aims to derive gravity necessarily out of relative inertia:
- "If I am successful, gravity would be understood as a direct and necessary consequence of the relativity of acceleration, the identity of the gravitational and inertial masses would be shown to be self-evident and the gravitational field would not only be equivalent to an accelerated space, as Einstein proposes, but gravity itself would be identified as a resistance to relative acceleration."

Chronology of (some of) the relevant papers

- 1872 - E. Mach: "History and root of the principle of conservation of energy."
- 1883 - E. Mach: "The science of mechanics"
- 1896 - Friedlaenders: "Absolute or relative motion"
- 1915 - H. Reissner: "On the possibility of deriving gravity as a direct consequence of the relativity of inertia"
- 1925 - E. Schroedinger: "On the possibility of the fulfillment of the relativity requirement in classical mechanics"
- 1953 - D. Sciama: "On the origin of inertia"
- 1961 - R. Dicke: "Dirac's cosmology and Mach's principle"
- 1969 - Sciama et al.: Generally covariant integral formulation of Einstein's field equations
- 1972 - J. Treder: The relativity of inertia.

Implementing Mach's principle: Inertia in rotating frames

- We begin by implementing **Mach's principle**: "Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. [...] The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first." (Mach, 1872)
- In order to do this we need to introduce two force fields which will appear in classically "non-inertial" frames:

$$\boldsymbol{E}_{cent} = \omega^2 \boldsymbol{r}, \qquad \boldsymbol{B}_{Cor} = -2\boldsymbol{\omega}. \tag{1}$$

Written in terms of potentials these are:

$$\Phi = -\frac{1}{2}(\boldsymbol{\omega} \times \boldsymbol{r})^2, \qquad \boldsymbol{A} = \boldsymbol{\omega} \times \boldsymbol{r}, \tag{2}$$

Implementing Mach's hypothesis: Source of the inertial potential

We can describe these two inertial forces in terms of a 4-vector potential:

$$A_\mu = (\Phi, \boldsymbol{A}), \tag{3}$$

- From a Newtonian perspective this A_μ would be interpreted as a property of absolute space
- In accordance with **Mach's hypothesis**, we seek its material origin in the masses of the universe
- This can be achieved by continuing a direct analogy with electromagnetism:[3]

$$A_\mu(x) = -k \int_V \frac{P_\mu(x')}{|x - x'|} dV \tag{4}$$

[3] Where $p_\mu = (\epsilon/c, \boldsymbol{p})$, with ϵ the energy density and $\boldsymbol{p}$ the linear momentum density

Implementing Reissner's hypothesis: Deriving gravity

We can now write our field equation:[4]

$$\Box A_\mu = -4\pi \frac{p_\mu}{c} \tag{5}$$

From here it is possible to derive the following expression for the gravinertial force in universe of homogeneous density ρ disturbed by a central point mass M_0 at $\boldsymbol{r} = \boldsymbol{r_0}$:[5]

$$\ddot{\boldsymbol{r}} \approx -\frac{c^2}{\Phi(\boldsymbol{r})} M_0 \frac{(\boldsymbol{r} - \boldsymbol{r_0})}{|\boldsymbol{r} - \boldsymbol{r_0}|^3} \tag{6}$$

Reissner's hypothesis then consists in the total identification of gravity with this dynamical residue of our relativized inertia:[6]

$$G = \frac{c^2}{\Phi(\boldsymbol{r})} \tag{7}$$

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[4] I thank my cllaborator Ophelliam Loiselet for helping me reconstruct this

[5] Where $\Phi(\boldsymbol{r})$ is the total scalar gravinertial potential of the universe

[6] First derived by Sciama (1953), analogues are found earlier in Schrödinger (1925); Reissner (1915)

Does Reissner's hypothesis imply a critical density cosmos?

- In principle, $G = c^2/\Phi$ is an experimentally testable result, indeed Sciama (1953) calculates: $\Phi = 2\pi\rho c^2 H^{-2}$ and finds decent agreement with experiment
- But such measurement results in cosmology are always only approximate
- Is there some other prediction that this leads to that is more directly testable?
- There is some strong suggestion that this condition is identical to the "critical density" condition in cosmology:[7]

"the Einstein-de Sitter model is the one where the total energy of the universe is zero, the kinetic energy and the negative gravitational potential energy just balancing. Well, if you think that kinetic energy manifesting inertia is due to gravitation, then you might intuit that the most Machian way of having one made by the other would be if there's equal amount of energy, which would give you uniquely the Einstein-de Sitter model, I still have a secret hope that that might turn out so" (Sciama, 1978)

[7] I thank my collaborator Antonis Antoniu for bringing my attention to the question of cosmic geometry

The flatness problem

Beyond the experimental evidence that has gather since the 2000's in favour of a flat or almost flat cosmos, there is an arguably stronger non-experimental reason to favour this result, this is due to the flatness problem:

- "This problem arises from the fact that, according to the standard world models, if the Universe were set up with a value of the density parameter differing even slightly from the critical value $\Omega_0 = 1$, it would diverge very rapidly from $\Omega_0 = 1$ at later epochs. There is nothing in the standard world models that would lead us to prefer any particular value of Ω_0. Why then is the density parameter close to one today?" (Longair, 2022)
- In standard cosmological models, no reason is given for this, the odds that a randomly generated universe would preserve this balance should be vanishingly small
- In contrast, if the result of Reissner's hypothesis indeed does correspond to the critical density condition, the preserve of this balance would be identical to the total identification of gravity with relative inertia

Recap

- The convergence of Mach's hypothesis and the equivalence hypothesis suggested that gravity may be the dynamical part of relative inertia
- Despite Einstein's Machian intentions, his theory implies the physical autonomy of spacetime and only embodies Mach's first hypothesis
- Reissner, a contemporary of Einstein, was the first to properly articulate the hypothesis that gravity may be a consequence of the Machian relativisation of inertia
- This idea is embodied in the expression $G\Phi = c^2$
- There is reason to believe that this result might be identical to the critical density condition in cosmology.
- In this case: the critical density cosmos would be the one in which gravity is entirely explicable in terms of relative inertia, and no further mass-interaction exists

Strengths of this hypothesis

- Explanation for the existence of gravity
- The "equivalence principle" is a consequence of the theory rather than a postulate
- Explains the value of G
- May explain observed flatness of cosmos
- May give a stable cosmos in which the critical density is ensured
- Remarkable elegance of these results: gravity is given a holistic role connecting local physics to large-scale structure such that the cosmos remains perfectly balanced

Question for the audience

Should science strive to explain natural phenomena, or merely describe them?

Thank You

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