On Reissner's hypothesis

Historical proposals for a Machian unification of gravity and inertia

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Contents

Relativity in subsystem and the universe

1. **Relativity of physical space:**
   - physics experiments may yield the same results whether done here or there, today or tomorrow, or if the lab moves at constant speed...
   - This is a generalisation of an empirical result, it is never perfectly applicable to reality (since subsystems are never closed), but applied in virtue of an idealisation
   - Key examples: (a) Galilean relativity in CM, (b) Poincaré-Einstein relativity in SRT
   - Not true under rotations since the environment can't be neglected.
2. **Relativity under "Leibniz shifts":**[1]
   - There is no difference in the relations of things, if the "whole universe" is shifted to the left by 5 meters, or boosted, or spun...
   - Analytic a priori — it is part of the definition of "whole universe", follows from PII
   - Should be true under any shape-preserving transformations, such as *rigid transformations*.

Note that physics is usually only concerned with (1) since this involves real motion.

[1] From the Leibniz-Clark correspondence.

Mach's principle

- Mach's principle has to do with this second sense (2) of relativity
- **For Newton:** the appearance of inertial forces in non-inertial frame was evidence of the reality of absolute space, which defines true motion
- Indeed, in order to properly define the notion of ***Force***, one needs to know what motion bodies will resort to in the absence of forces — we need a **law of inertia**
- Classical Mechanics is based on this paradigmatic distinction between *caused* and *uncaused* motion.
- **For Mach** ,as I argued in Fay (2024): this distinction is illusory, "inertial" motion is just as *caused* as "forced" motion, but it is caused by some homogeneous aspect of the environment
- Once that environment is taken into account, the relativity principle should hold under arbitrary rigid transformations of the reference frame.

Mach's two hypotheses

- Now Mach's principle itself is agnostic about what the actual cause of inertial motion is,
- Indeed Mach proposes two distinct hypotheses in his works:

Mach's principle:
Inertia is required to have a physical origin — *what is it?*

Mach hypothesis 1:
Space is a physical medium causally responsible for inertia

Mach hypothesis 2:
Physical space is an "abbreviated reference" to masses — which are the sources of inertial influences

Is GR Machian?

- General Relativity (GR) — the current paradigmatic theory of space, time and inertia, can be seen as Machian in the sense of **MH1**
- However it is kind of a hybrid between **MH1** and **MH2** since it also includes *inertial induction* effects such as the Lense-Thirring effect

Einstein's equivalence hypothesis

- Although Einstein did not produce a theory which successfully implemented **MH2**, this second hypothesis of Mach was nonetheless crucial to Einstein's reasoning on his way to GR
- Einstein referred to this hypothesis as "Mach's principle" though I've argued this is a misnomer
- Einstein's key contribution for our purposes today was the "equivalence principle", which should also really be called the **equivalence hypothesis**.
- This comes from the observation that all objects fall with the same acceleration in a gravitational field $m_g = m_i$
- Just like inertial forces, gravity is a *universal force*[2] — this suggests the possible **unification** of gravity and inertia

[2] Reichenbach's term

Two converging hypotheses

Mach hypothesis 2:
Inertial motion is conditioned by the masses of the cosmos

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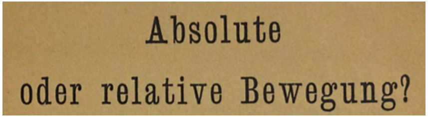
- The physical properties of "space" constraining the motion of bodies are due to an interaction with *other bodies*, and this interaction is mediated through the force which we call "gravity".

First hint: Immanuel Friedlaender's footnote

Figure 1: Immanuel Friedlaender


Absolute
oder relative Bewegung?

- Although Reissner's hypothesis is only conceivable once we have taken on board the significance of the **EH**, the first mention of an idea along these lines occured in 1896.
- "However, it seems to me that the correct form of the law of inertia will only then have been found when relative inertia as an effect of masses on each other and gravitation, which is also an effect of masses on each other, have been derived on the basis of a unified law." (Friedlaender and Friedlaender, 1896)[a].

[a] See Barbour and Pfister (1995) and Renn (2007) for English translations

Why have I called this Reissner's hypothesis?

- Between 1913 and 1915, Hans Reissner is closely following Einstein's pioneering work on gravity, in the second of two papers, titled: **"On the possibility of deriving gravity as the direct consequence of the relativity of inertia"**, he articulates clearly what I am calling "Reissner's hypothesis".
- First, concerning the **EH**: "Mr. Einstein's equivalence hypothesis which asserts the mechanical and optical identity of an acceleration field with a field of constant gravity seems to imply the deeper meaning that gravity is also a resistance to acceleration."
- Reissner aims to derive gravity necessarily out of relative inertia: "If I am successful, gravity would be understood as a direct and necessary consequence of the relativity of acceleration, the identity of the gravitational and inertial masses would be shown to be self-evident and the gravitational field would not only be equivalent to an accelerated space, as Einstein proposes, but gravity itself would be identified as a resistance to relative acceleration."

Who was Hans Reissner?

Figure 2: Hans and Josephine Reissner

- Pioneering German aeronautical engineer
- First person to build an all-metal aircraft (Reissner Canard)
- First to derive 'Reissner-Nördstrom metric' (Reissner, 1916)
- Articulated *Reissner's hypothesis* concerning the origin of gravitation (Reissner, 1915)

Some of the papers implementing this

Over the course of the 20th Century, various papers have been published that to some degree embody this idea of Reissner's.

- 1915 - H. Reissner: "On the possibility of deriving gravity as a direct consequence of the relativity of inertia"
- 1925 - E. Schrödinger: "On the possibility of the fulfillment of the relativity requirement in classical mechanics"
- 1953 - D. Sciama: "On the origin of inertia"
- 1972 - J. Treder: "The relativity of inertia"
- 1975 - J. Barbour: "Forceless machian dynamics"

Unfortunately, apart from Reissner's paper, this is usually not expressed as an explanation for the existence of gravity.

Implementing **Mach's principle**: Inertia in rotating frames

- We begin by implementing **Mach's principle**: "Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. [...] The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first." (Mach, 1872)
- In order to implement this we need to introduce two force fields which will appear in classically "non-inertial" frames:

$$\boldsymbol{E}_{cent} = \omega^2 \boldsymbol{r}, \qquad \boldsymbol{B}_{Cor} = -2\boldsymbol{\omega}. \tag{1}$$

Written in terms of potentials these are:

$$\Phi = -\frac{1}{2}(\boldsymbol{\omega} \times \boldsymbol{r})^2, \qquad \boldsymbol{A} = \boldsymbol{\omega} \times \boldsymbol{r}, \tag{2}$$

Implementing **Mach hypothesis 2**: Source of the inertial potential

We can describe these inertial forces in terms of a 4-vector potential: $A_\mu = (\gamma_A \Phi, \gamma_A \boldsymbol{A})$.

- From a Newtonian perspective this A_μ would be interpreted as a property of absolute space
- In accordance with **MH2**, we seek its material origin in the masses of the universe
- This can be achieved by directly continuing the analogy with electromagnetism:

$$A_\mu(r) = -k \int_U \frac{P_\mu(t - |r - r'|/c, r')}{|r - r'|} d^3 r' \qquad \Box A_\mu = -4\pi k P_\mu \tag{3}$$

with $P_\mu = (\gamma \rho c, \gamma \rho \boldsymbol{v})$ being the four momentum density throughout the universe.

Finding the coupling strength k

- The strength k of the couping of A^μ to the four-momentum density P^μ is determined by the requirement that A^μ produce exactly the right relativistic centrifugal and Coriolis forces.
- To do this we begin by considering a homogeneous universe of density ρ in a frame in which it is rotating with angular velocity ω.
- In cylindrical coordinates, the four-momentum distribution will be given by $P_\mu = (\gamma_\omega \rho c, \gamma_\omega \rho \omega r \hat{\theta})$
- This is integrated to get the four-vector potential $A_\mu = -K\gamma_\omega(c, \omega r\hat{\theta})$, where:

$$K = k \int_U \frac{\rho(t - |r - r'|/c, r')}{|r - r'|} d^3 r'. \tag{4}$$

Finding the coupling strength k (part 2)

- Now, by decomposing the 4-vector potential $A_\mu = (A_0, \bar{A})$, we can calculate the centrifugal and Coriolis fields (E and B respectively):

$$\bar{E} = -\nabla A_0 - \partial_t \bar{A} \qquad \bar{B} = -\nabla \times \bar{A} \tag{5}$$

$$\bar{E} = -\partial_r A_0 \hat{r} \qquad \bar{B} = -\frac{1}{r}\partial_r (r|\bar{A}|)\hat{z} \tag{6}$$

$$\bar{E} = Kc\partial_r \gamma_\omega \hat{r} \qquad \bar{B} = -K\frac{1}{r}\partial_r (r\gamma_\omega \omega r^2)\hat{z} \tag{7}$$

$$\bar{E} = \frac{K}{c}\gamma_\omega^3 \omega^2 r\hat{r} \qquad \bar{B} = -K(\gamma_\omega^3 \beta_\omega^2 \frac{\omega}{c} + 2\gamma_\omega \frac{\omega}{c})\hat{z} \tag{8}$$

where $\beta_\omega = \frac{\omega r}{c}$ and $\gamma_\omega = (1 - \beta_\omega^2)^{-1/2}$.

Finding the coupling strength k (part 3)

- Now, we consider a test particle which is commoving with the rotating universe, so that it's 4-momentum is similarly: $P_\mu = m_0\gamma_\omega(c, \omega r\hat{\theta})$. The force on this particle can therefore be calculated by the analogue of the Lorentz force law:

$$\bar{F} = m_0 c\gamma_\omega \bar{E} + m_0 c\gamma_\omega \beta_\omega \hat{\theta} \times \bar{B} \tag{9}$$

$$= K m_0 \omega^2 r \left[\gamma_\omega^4 - \gamma_\omega^4 \beta_\omega^2 - 2\gamma_\omega^2\right] \hat{r} \tag{10}$$

$$= K m_0 \omega^2 r \gamma_\omega^4 \left[1 - \beta_\omega^2 - 2\gamma_\omega^{-2}\right] \hat{r} \tag{11}$$

$$= K m_0 \omega^2 r \gamma_\omega^4 \left[\gamma_\omega^{-2} - 2\gamma_\omega^{-2}\right] \hat{r} \tag{12}$$

$$= -K m_0 \omega^2 r \gamma_\omega^2 \hat{r} \tag{13}$$

- This will produce a four-acceleration or **proper acceleration** $\bar{a}_r$:

$$\bar{a}_r = -K\gamma_\omega^2 \omega^2 r\hat{r}. \tag{14}$$

Finding the coupling strength k (part 4)

- The proper acceleration $\bar{a}_r = -K\gamma_\omega^2\omega^2 r\hat{r}$ we derived exactly matches the expected proper acceleration provided that $K = 1$
- Finally, this tells us that the coupling strength k is given by:

$$k(r) = \left(\int_U \frac{\rho(t - |r - r'|/c, r')}{|r - r'|} d^3 r'\right)^{-1}, \tag{15}$$

- We can simply write this as $k = \Phi^{-1}$, so that finally, our field equation becomes:

$$\Box A_\mu = -4\pi\Phi^{-1} P_\mu \tag{16}$$

...which is not a 'normal' field equation, since the coupling Φ introduces a dependence the broader cosmic structure.

Implementing Reissner's hypothesis: Deriving gravity

- From here we examine what happens to this relativised inertial law in an idealised cosmos which is homogeneous of density ρ apart from a single mass M
- In this case the scalar and vector potentials will be given by:

$$A_0 = -kc\left(\Phi_U + \frac{M}{r}\right) \qquad \bar{A} = -k\bar{v}\left(\Phi_U + \frac{M}{r}\right) \tag{17}$$

where Φ_U is the gravitational potential of the universe of density ρ discounting the contribution of the mass M.

- Since the universe in this scenario is not rotating, the vorticity is zero, so we only need to consider $\bar{E}$, which is given by:

$$E = -\nabla A_0 - \frac{1}{c}\partial_t \bar{A} = kc\frac{M}{r^2}\hat{r} + \frac{k}{c}\bar{a}\left(\Phi_U + \frac{M}{r}\right) \tag{18}$$

Implementing Reissner's hypothesis: Deriving gravity (part 2)

- There are now two approaches to deriving the acceleration of test particle: (1) We either consider the acceleration induced by $\bar{E}$ in our frame in which the universe is stationary so that $\bar{A} = 0$, or (2) we consider how the universe moves ($\bar{A}$) with respect to an "inertial" frame, i.e. where $\bar{E} = 0$.
- Interestingly, Sciama (1953) uses (2) the inertial frame method, which does not require us to have calculated the coupling strength k beforehand.
- However, either method yields the same result:

$$\bar{a}_p = \frac{c^2}{\Phi}\frac{M}{r^2}\hat{r} \tag{19}$$

- This is just Newton's law of gravity provided that $G\Phi = c^2$.
- Thus, if this equation holds, we will have explained gravity entirely as a phenomenon of relative inertia, this is **Reissner's hypothesis**.

How can we verify Reissner's hypothesis?

This proposed *gravinertial* unification bears some analogies to Maxwell's unification of electricity, magnetism and optics:

$$c^2 = \frac{1}{\mu_0 \epsilon_0} \qquad\qquad G = \frac{c^2}{\Phi(\boldsymbol{r})}$$

- In the case of Maxwell, μ_0, ϵ_0 and c were already known to reasonably good precision in the 1860s.
- However for us Φ remains challenging to calculate
- Nonetheless, we can investigate whether $G\Phi = c^2$ might imply further theoretical results that may be tested indirectly
- I will look at 3 candidates: (1) Jordan's zero-energy condition, (2) Dirac's large number coincidences, (3) the critical density cosmos.[3]

[3] This will mostly be based off an ongoing correspondence with Alex Blum.

A zero-energy universe

- It's not hard to notice that the condition $G\Phi = c^2$ immediately tells us that the total (negative) gravitational potential a particle has with respect to the rest of the universe is fully and exactly compensated by its rest mass energy.
- In fact, this equation was derived, totally independently of considerations of Mach's principle in 1939 by Pascal Jordan:
- "We interpret [this equation] as the expression of the energy principle [...] it means the added potential energies mc^2 of all material particles are precisely compensated by the negative gravitational energy, so that the whole universe energy stays constant (namely virtually null)." (Jordan, 1939)
- Why this zero-energy universe should at the same time be the Machian universe remains quite mysterious to me, I've been talking with Alex Blum about this lately; I might say more in Q&A.

Dirac's numbers

- Jordan's 1939 paper was in large part motivated by a paper by Dirac from the previous year in which he points to certain numerical coincidences
- Jordan develops this theory further: "Jordantherorie"
- Taken up by Robert Dicke and Carl Brans who combine this with their understanding of Mach's principle and Sciama's result to develop "Brans-Dicke theory"
- Both Wheeler and Dicke at various points use the Lense-Thirring effect to derive a similar approximate condition on G:

$$\frac{GM_U}{c^2 R_U} \sim 1 \tag{20}$$

Critical density

- From a recent book 2022 "Cosmic Relativity" by C.S. Unnikrishnan:
- "The critical density $\rho c = 3H_0^2/8\pi G$ that corresponds to a spatially flat Universe has a Newtonian interpretation that it is the energy density at which the kinetic energy of a particle is exactly equal to its negative gravitational potential energy in the Universe: $mv^2/2 = m\dot{r}^2/2 = 4\pi G\rho r^2/3$. Then, $\rho = 3\dot{r}^2/8\pi G r^2 = 3H^2/8\pi G$, which is the critical density. This can be interpreted as the natural constraint of energy conservation in the creation of the matter in the Universe from vacuum, from nothing. Then, the total energy before and after creation has to be zero. This is achieved when the Universe has the critical density at any epoch. Also, it fixes the magnitude of the gravitational potential as $\Phi_u = -c^2$; only then the rest mass energy $E = mc^2$ is matched by an exactly equal and negative gravitational potential energy $E_g = mc^2$ making the total zero."
- I've never seen a rigorous proof of this, so I'm currently trying to get in touch with Unnikrishnan to find out why he thinks this is the case

Recap

- whereas for subsystems of the universe symmetries and the relativity principle arise as idealisations of convenient empirical regularities, for the universe as a whole it is analytic.
- This leads us to **Mach's principle** — inertial motion is not motion in the absence of causes but motion who's cause lies in some regularity in the environment.
- As a candidates for the cause of inertial motion, Mach suggests **MH2**: the distant stars condition the inertial frame.
- In combination with **EH**, this suggests **RH**: Gravity is the dynamical residue of the relativised inertial law.
- This idea can be realised in a theory which treats gravinertia by analogy with electromagnetism, leading to the relation: $G\Phi = c^2$.
- The result suggests curious cosmological implications, including a zero-energy universe and a prediction of the critical density condition.

Thank You

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