On Reissner's hypothesis

Historical proposals for a relational unification of gravity and inertia
In Celebration of the 111th anniversary of Reissner's paper

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Contents

Mach against classical causation

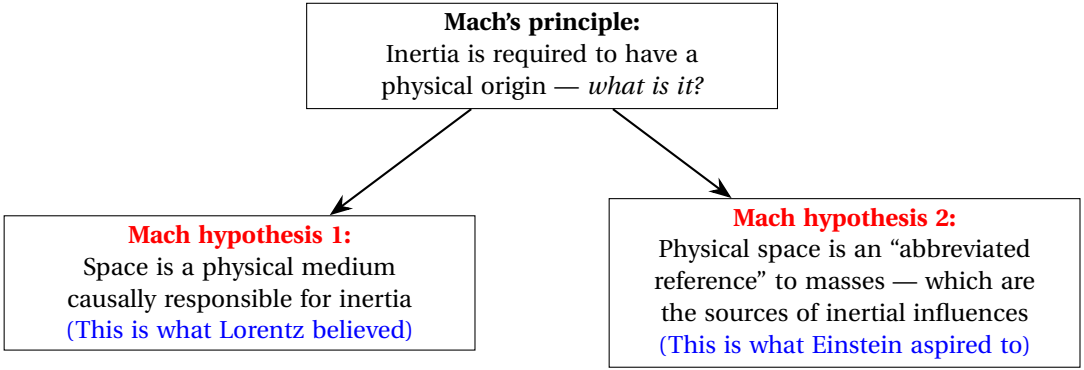
What is "Mach's principle"?

- In essence, it is a critique of the classical conception of causation (Fay, 2024).
- Look especially at Mach (1872) on "conservation of energy".
- In classical mechanics, causes are identified with forces, which produce accelerations in bodies.
- This system necessitates that something define a body's motion in the absence of causes.
- **For Newton**, this is "Absolute Space" and "Absolute Time".
- **For Mach**, as for others, space and time are conceptual constructions, moreover, nothing happens in the absence of causes: the roughly homogeneous "inertial motion" needs some causes in some homogeneous feature of the environment:

$$L = \frac{1}{2}mv^2 \longrightarrow ?$$

Mach's two hypotheses

- Now Mach's principle itself is agnostic about what the actual cause of inertial motion is,
- Indeed Mach proposes two distinct hypotheses in his works (Mach, 1872, 1883):



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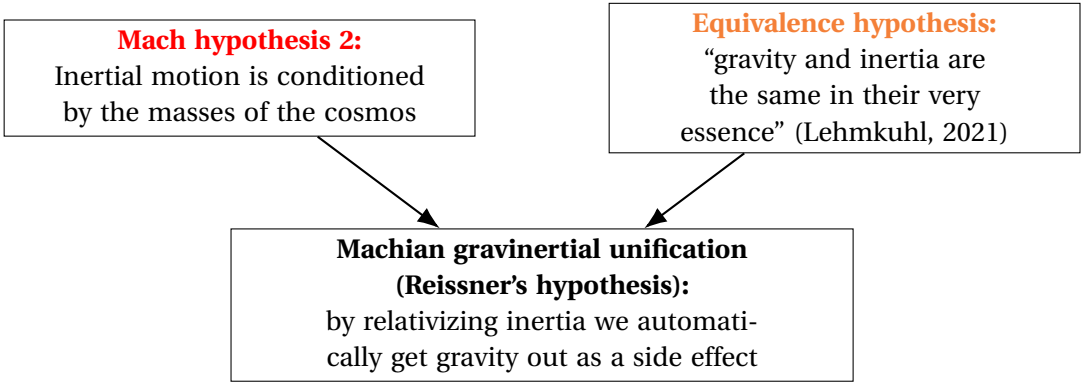

- Reissner (1915): "If I am successful, **gravity** would be understood as a **direct and necessary consequence of the relativity of acceleration**, the identity of the gravitational and inertial masses would be shown to be self-evident [...]"

Who was Hans Reissner?

Figure 1: Hans and Josephine Reissner

- Pioneering German aeronautical engineer
- First person to build an all-metal aircraft (Reissner Canard)
- First to derive 'Reissner-Nördstrom metric' (Reissner, 1916)
- Articulated *Reissner's hypothesis* concerning the origin of gravitation (Reissner, 1915)

Difficulties with applying these ideas

- Gravity should have a finite propagation speed, we need to account for retarded action, or build a field theory...
- This leads to apparent circularity: gravity modifies the light cone structure along which you integrate.
- Weyl (1924) for instance points out that the dynamical conception of spacetime undermines the idea that you can even have a notion of relative motion at a distance.
- For these reasons, most attempts to implement these ideas have been in the context of classical mechanics.
- We will put aside these problems for now, and examine the various classical models to see what this research program has to offer.

Chronology

We will cover most of these key 20th C. classical Machian models:[1]

- 1915 — H. Reissner: "On the possibility of deriving gravity as the direct consequence of the relativity of inertia"
- 1925 — E. Schrödinger: "On the possibility of the fulfillment of the relativity requirement in classical mechanics"
- 1953 — D. Sciama: "On the origin of inertia"
- 1955 — G.B. Brown: "A theory of action-at-a-distance"
- 1972 — H.J. Treder: "The relativity of inertia"
- 1975 — J. Barbour: "Forceless Machian dynamics"
- 1976 — R.J. Cook: "Is gravitation a result of Mach's inertial interaction"
- 1989 — Assis: "On Mach's principle"

[1] Special thanks to Dennis Braun, this section is based on our collab paper (Fay and Braun, 2025)

The Weber potential

- The Lagrangian for inertial and gravitational motion in classical mechanics is:

$$L_i = \frac{1}{2} m_i v_i^2 - m_i G \sum_{j \neq i} \frac{m_j}{r_{ij}}, \tag{1}$$

- The problem here is that the kinetic term is defined w.r.t. absolute space.
- In order to relativise it, one option that immediately springs to mind is to introduce a velocity dependent term to the gravitational potential, just like Weber's law in electromagnetism, which is relativised.
- "However, it seems to me that the correct form of the law of inertia will only then have been found when relative inertia as an effect of masses on each other and gravitation, which is also an effect of masses on each other, have been derived on the basis of a unified law. [...] For this it would be very desirable to resolve the question of whether Weber's law applies to gravity" — Friedlaender (1896)

Schrödinger and Assis

- Schrödinger (1925) was the first to do this, he proposed the following interaction Lagrangian, Assis (1989) would later independently rediscover the same theory.

$$L_{ij} = \delta \frac{m_i m_j}{r_{ij} c^2} \dot{r}_{ij}^2 - \frac{m_i m_j}{r_{ij}}, \tag{2}$$

- This form is analogous to the Weber potential for electromagnetism.
- Although $\delta \sim 1$ is expected, by fitting to the known perihelion shift of Mercury, Schrödinger calculates $\delta = 3$ and then, taking $\Phi_i = \sum_j m_j / rij$:

$$G_i = \frac{c^2}{2\Phi_i} \tag{3}$$

- This is a remarkable result for 1925 as it implies a universe much larger than was known, & Hubble's discovery of other galaxies had only been made the year before.
- In fact, eq. 3 is the same order of magnitude as the critical density condition.

The anisotropy controversy

- These "Machian" models lead to a tensorial expression for inertial mass, i.e. it is anisotropic.
- Null results (Hughes et al., 1960; Drever, 1961) of mass anisotropy have been the main reason for their rejection historically.
- There are two responses to this:
  1. Dicke (1961) has argued that anisotropy effects may be unobservable due to covariance of measuring apparatus (Reissner also speculated about this in 1915).
  2. Treder (1972) proposed a modification of the Lagrangian $\dot{r}_{ij} \to v_{ij}$ to avoid this issue altogether.

$$L_{ij} = \delta \frac{m_i m_j}{c^2 r_{ij}} \mathbf{v}_{ij}^2 - \frac{m_i m_j}{r_{ij}}, \tag{4}$$

- This question could certainly be resolved by further attention from physicists.

Sciama's model: "On the origin of inertia" — link to SR?

- Interesting because he gets similar results from a field-theory approach, though the model is incomplete.
- Instead of an analogy with Weber's EM, Sciama (1953) uses and analogy with Maxwell's EM.

$$\Phi = -\int_V \frac{\rho}{r}\, dV, \qquad \mathbf{A} = -\int_V \frac{\mathbf{v}\rho}{cr}\, dV, \tag{5}$$

- The model is invariant under linear accelerations provided that $G\Phi = c^2$.
- For rotation invariance, Sciama gets the appropriate relational Coriolis force from $\mathbf{A}$, however for the centrifugal force he has to appeal to a modification of Φ due to $A^\mu = (\Phi, \mathbf{A})$ being a 4-vector.

$$\Phi = -1 \longrightarrow \Phi' = -(1+\omega^2 r^2)^{1/2}$$

- This reasoning is flawed since, Sciama should have defined Φ such that the transformation is induced by the motion of the sources, for instance $\Phi = -\int \frac{\gamma\rho}{r}\, dV$
- Suggests that if developed properly, contact with SR may entail $G = c^2/\Phi$

The Reissner-Barbour-Cook mechanism

- If we think in 4-vector notation, Sciama's model does not involve positing gravity and inertia separately.
- However it has also been proposed that you can get gravity out of a purely inertial theory from a 3-vector interaction.

$$L_i = \frac{1}{2} m_i v^2 \longrightarrow L_i = \frac{1}{2} \mu_i \sum_j \frac{\mu_j \dot{r}_{ij}^2}{r_{ij}}$$

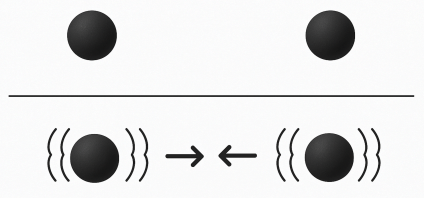

Figure 2: Vibration-induced attraction

Reissner and Barbour

- This mechanism was discovered 3 times totally independently by Reissner (1915); Barbour (1975); Cook (1976).
- Reissner: **Postulate 2:** Weight, or Newtonian gravitation, can be represented as an inertial force arising from the rotation of mass particles relative to each other. This leads to an expression for the gravitational constant: $G = 3\frac{k_i^2\omega_i^2 + k_j^2\omega_j^2}{\phi_i\phi_j}$
- Barbour: "The most obvious possibility that springs to mind is that the rest-mass energy of the particles is associated with an incessant internal motion at velocity c (all energy then being kinetic in origin) or alternatively with the zitterbewegung of elementary particles [...]"
- From this he gets an expression of the form $G \sim c^2/\Phi$, with $\Phi = R_u/M_u$.

Cook's zitterbewegung: "Is gravitation a result of Machs inertial interaction."

- Without knowing of Barbour's paper, Cook (1976) does exactly this.
- He first considers Sciama (1969)'s expression for linear acceleration induction:

$$\mathbf{F} = \frac{GmM\mathbf{a}}{c^2 r} \tag{6}$$

- This gives invariance of dynamics under linear accelerations if $G = c^2/\Phi$
- If the source of inertial influences is vibrating isotropically with some velocity v_i, the time-averaged force comes out as:

$$\langle F^i \rangle = -\frac{Gm_1 m_2 r_2^i}{r_2^3} \frac{\langle v_1^2 \rangle}{3c^2} \tag{7}$$

- By identifying this vibration with the "zitterbewegung" of Dirac fermions, Cook uses the (well known) result $\langle v_1^2 \rangle = 3c^2$ to recover Newton's law.
- This suggests a hidden harmony between SR and QM.

Barbour's 1975 program

> *"The ideal would therefore be a theory based on a generalization of [the Lagrangian]* ***in which special relativity and quantum mechanics play an integral part****, providing a natural framework for the necessary internal motion. In such a theory,* ***the four universals of Nature—gravity, inertia, special relativity and quantum mechanics—would clearly be linked in an intimate and inseparable manner****" (Barbour, 1975)*

Although Barbour abandoned this class of theories due to the anisotropy issue, I think that the results seem promising.

Links to cosmology

The Machian program is inherently cosmological. Two connections stand out:

1. The critical density condition derived from the Friedmann equations gives: $G\Phi = \frac{3}{4}c^2$, which is of the same order as all the expressions we saw.
   - Sciama had a preference for critical models throughout his career due to this connection (Antoniou and Fay, 2025).
2. Sciama points out that his expression $G\Phi = c^2$, "implies that the total energy (inertial plus gravitational) of a particle at rest in the universe is zero."

$$E_p = -m_p G\Phi_p + m_p c^2 = 0 \tag{8}$$

   - Similar constraints on G were discussed for this purpose by Haas (1936) and Jordan (1939) independently of considerations of Mach's principle.[2]
   - In the context of Mach's principle, it is reminiscent of the result that total linear and angular momentum of the universe are zero.

[2] I thank Alex Blum for pointing me to the Jordan connection.

Summary

- Mach's principle: critique of classical causality — we need a cause of inertial motion
- Mach's hypothesis: inertial motion might be cause by the masses of the universe
- Reissner's hypothesis: relativised inertia may directly give rise to gravity
- Analogies with electromagnetism were employed by Schrödinger, Assis, Treder and Sciama
- Reissner, Barbour and Cook derive gravity due to particle vibration
- There appears to be a harmony between Sciama's model which connects with SR and Cook's model which connects with QM
- The resulting constraint $G\Phi = c^2$ seems to also be in harmony with certain ideas and results in cosmology.

Thank You

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